$$\therefore\ x=\frac{\sqrt{(c^4-a^2b^2)}+c\sqrt{(a^2-b^2)}}{\sqrt{(c^2-b^2)}},\ y=\frac{\sqrt{(c^4-a^2b^2)}-c\sqrt{(a^2-b^2)}}{\sqrt{(c^2-b^2)}}.$$

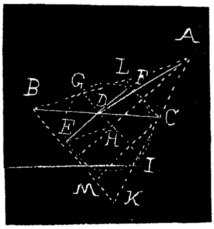
Substituting numbers we get $\cos\theta=\frac{21}{29}$, $\theta=43^\circ\ 36'\ 9''$.

$$x=\frac{\sqrt{(589)}+5\sqrt{(5)}}{\sqrt{(21)}}=7.73575 \text{ feet},\ y=\frac{\sqrt{(589)}-5\sqrt{(5)}}{\sqrt{(21)}}=2.85625 \text{ feet}.$$

$z=\sqrt{(29)}=5.38516$ feet.

IV. **Solution by A. H. BELL, Hillsboro, Ill.**

Let the vertical plane be ABK, A the vertex and BK the base of a right cone. The horizontal cutting plane BC is the major axis of the ellipse with D the projection of the minor axis, the cutting plane GI passing through D, and parallel to the base BK, is a circle and contains the minor axis of the ellipse. Revolving the circle 90° with the diameter GI as an axis, the chord EF is the minor axis of the ellipse; and s, s' are the foci. LC is a circle and parallel to the base BK of the cone.



$BD=DC=a=3$ feet, $ED=DF=b=2$ feet, $AD=c=5$ feet.

The properties of an ellipse give $s,\ s'=BL=CK$......(1).

$BK\times CL=EF^2=4b^2$..... (2).

$BC^2=BL^2+BK\times CL$. $\therefore$ $BL=2(a^2-b^2)^{\frac{1}{2}}=4.4721360$......(3).

In the right triangle ADF, $AF=AI=AG=(b^2+c^2)^{\frac{1}{2}}=\sqrt{(29)}=5.3851648$.

$BG=GL=CI=\frac{1}{2}BL=2.2360680$.

$AC=AI-CI$. $AB=AI+CI=7.6212328$, $AC=3.1490968$, and the point A is determined.

NOTE. Radius, $GH=(\frac{29}{6})^{\frac{1}{2}}=2.198484326+=\left(\frac{b^2(b^2+c^2)}{2b^2+c^2-a^2}\right)^{\frac{1}{2}}$.

$DH=(GH^2-b^2)^{\frac{1}{2}}=(\frac{5}{6})^{\frac{1}{2}}=0.9128709$.

75. Proposed by J. C. NAGLE, A. M., M. C. E., Professor of Civil Engineering, State Agricultural and Mechanical College, College Station, Texas.

The water tank at the Nacogdoches River on the H. E. & W. T. Ry. is filled by a 3-inch pipe from a reservoir in which the water level is 6 feet above water in tank when full. The top diameter of tank is 17 feet, the bottom diameter is 19 feet, 8 inches, and the pipe projects 10 inches through the bottom. The depth is 13 feet, 6 inches. Find the time required to fill tank, taking the pipe as clean and free from sharp bends, except the right-angled one directly under tank. This bend is 12 feet below outlet of pipe, so that the total length of pipe is 1972 feet. Compare the result with the time of filling if the inlet pipe projected over top of tank instead of entering at the bottom.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Altitude of cone with top of tank as base$=86\frac{1}{16}$ feet; altitude of cone with bottom of tank as base$=99\frac{9}{16}$ feet; altitude of cone with section 10 inches from bottom as base$=98\frac{35}{48}$ feet.

V=volume of tank$=\pi\{[(59)^3-(51)^3]/(4)^3\}=1136.375\pi$ cubic feet; V_1=volume of portion 10 inches from bottom$=\pi\{[(4779)^3-(4739)^3]/(324)^3\}=79.90614551\pi$ cubic feet. $V_2=V-V_1=1056.46885449$ cubic feet.

Let v=velocity of discharge in feet per second, d=diameter of pipe in feet, l=length, h=head of water, W=weight of water in pounds discharged per second, f=coefficient of friction, β=coefficient of resistance at entrance of pipe, δ=coefficient of contraction at elbow.

Then $h=(v^2/2g)[(fl/d)+\beta+\delta+1]$. (Hydromechanics).

$\therefore v=\sqrt{\{2ghd/[fl+d(\beta+\delta+1)]\}}$.

$g=32.16$, $l=1972$, $d=\frac{1}{4}$, $f=.030268$, $\beta=.505$, $h=18\frac{3}{8}$, $\delta=.9846$.

$\therefore v=.516351\sqrt{h}$. $Q=\frac{1}{4}\pi d^2v=.034858\pi$ cubic feet, quantity discharged per second.

$W=62\frac{1}{2}Q=MQ$.

If n is the distance of the top of the tank above the outflow, the resistance or pressure of water in tank on outflow

$$=\tfrac{1}{2}M(\tfrac{1}{64}\pi)\int_0^n dx=\frac{\pi Mn}{128}.$$

$$\therefore hW=\frac{v_1^2}{2g}\left[\left(\frac{fl}{d}+\beta+\delta+1\right)W+\frac{\pi Mm}{128}\right].$$

$$\therefore 2gh=v_1^2\left(\frac{fl}{d}+\beta+\delta+1+\frac{\pi n}{128Q}\right).$$

$\therefore v_1=2.21788$, since $n=12\frac{2}{3}$. $\therefore Q_1=\frac{1}{4}\pi d^2v_1=.034654\pi$ cubic feet per second.

T=time$=(V_1/Q)+(V_2/Q_1)=(79.90614551\pi/.034858\pi)+(1056.46885449\pi/.034654\pi)$.

$\therefore T=32778.53$ seconds=9 hours, 6 minutes, 18.53 seconds.

If filled from the top we will not consider the bend, but suppose pipe 1960 feet long and $h=6$ feet.

Then $h=(v_2^2/2g)[(fl/d)+\beta+1]$.

$\therefore v=\sqrt{\{2ghd/[fl+d(\beta+1)]\}}=1.2713$.

$\therefore Q_2=\frac{1}{4}\pi d^2v_2=.01986\pi$ cubic feet.

t=time$=(V/Q_2)=1136.375\pi/.01986\pi=57219.3$ seconds.

$\therefore t=15$ hours, 53 minutes, 39.3 seconds. $t-T=6$ hours, 47 minutes, 20.77 seconds.

II. Solution by P. H. PHILBRICK, M.S., C.E., Chief Engineer for Kansas City, Watkins & Gulf Railway Co., Lake Charles. La.

First. To find the time of filling the lower 10 inches of the tank. The head is $h=13.5+6-\frac{5}{6}=18\frac{2}{3}$ feet. Let v=velocity at the outlet, and s=the area of the cross-section of the pipe$=(\frac{1}{4}\pi)(\frac{1}{4})^2=.0491$.

Then $h=v^2/2g+m(v^2/2g)$, or $v=[2gh/(1+m)]^{\frac{1}{2}}$ ......(1), in which m is the sum of the resistances to the entrance and movement of water in the pipe.

The diameter of the tank 10 inches from the bottom is 19.5 feet. Hence the volume of the lower 10 inches of the tank$=(\frac{1}{12}\pi)[(19.67)^2+(19.5)^2+19.67\times19.5]=V$......(2).

Then from (1) and (2), $t=V/vs$..... (3).

According to *Bowser*, Articles 110 and 112, m may be taken$=.03\times1972\div\frac{1}{4}+1.5+.98=238.48$.

Second. To find the time of filling the remainder of the tank. Let x be the height in feet of the top of the reservoir above the surface of the water at the time t. The diameter of the tank at the surface of the water is $17+(\frac{16}{81})(x-6)=15.815+.1975x$. Hence the area is $A=(\frac{1}{4}\pi)(15.815+.1975x)^2=196.4+4.906x+.0306x^2$. Let dx be the rise in the water in the time dt. Then Adx is volume of water admited in time dt ....(4).

We also have, as shown above, $v=[2gx/(1+m)]^{\frac{1}{2}}$, and, therefore, the volume of water admitted in time dt is equal to $[2gh/(1+m)]^{\frac{1}{2}}\times sdt$......(5).

From (4) and (5) we have, $dt=[(1+m)/2g]^{\frac{1}{2}}\times1/s=[(196.4+4.906x+.0306x^2)dx]/x^{\frac{1}{2}}$ .....(6).

Integrating between $x=4=18\frac{2}{3}$ and $x=h=6$, we have $t'=[(1+m)/2g]^{\frac{1}{2}}\times.0491[392.8(H^{\frac{1}{2}}-h^{\frac{1}{2}})+3.271(H^{\frac{3}{2}}-h^{\frac{3}{2}})+0122(H^{\frac{5}{2}}-h^{\frac{5}{2}})$......(7).

$t+t'$=the total time required to fill the tank.

Third. If the inlet pipe projected over the top of the tank, we would have as in equation (1), $v_1=[2gh_1/(1+m)]^{\frac{1}{2}}$......(8), in which $h_1=6$ feet.

Also volume of tank is $V=(\frac{1}{4}\pi)[(19.67)^2+17^2+17\times19.67]$......(9).

Then $T=V/v_1s$......(10).

76. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Show that

$$\log[x-a-b\sqrt{(-1)}]=\tfrac{1}{2}\log[(x-a)^2+b^2]-\sqrt{(-1)}\tan^{-1}\frac{b}{x-a},$$

Naperian logarithms being used.

I. Solution by J. O. MAHONEY. B. E., M. Sc., Central High School, Dallas. Tex.; F. ANDEREGG, A. M., Oberlin College, Oberlin, O.; ARTHUR C. LUNN, University of Chicago, Chicago, Ill.; COOPER D. SCHMITT, A.M., University of Tennessee, Knoxville, Tenn.; BURKE SMITH. University of Washington, Seattle, Wash.; H. C. WHITAKER, A. M., Ph. D., Manual Training School, Philadelphia, Pa.

Let $(x-a)-ib=r(\cos\phi-i\sin\phi)$.

Then $r^2=(x-a)^2+b^2$, $\tan\phi=b/(x-a)$.

Thus $\log[x-a-ib]=\log[r(\cos\phi-i\sin\phi)=\log r+\log e^{-i\phi}=\log r-i\phi$

$=\frac{1}{2}\log[(x-a)^2+b^2-i\tan^{-1}[b/(x-a)]$.